J. Quant. Spectrosc. Radiat. Transfer. Vol. 13, pp. 79–81. Pergamon Press 1973. Printed in Great Britain

THE APPLICABILITY OF AN APPROXIMATE EXPRESSION FOR RADIATIVE HEATING*

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(*Received* 15 *June* 1972)

Abstract—An analysis considering small departures from radiative equilibrium within a gas, for which radiative heating is approximately formulated in terms of a radiative response time, is compared with an exact solution. It is shown that the approximate formulation does not properly describe local departures from radiative equilibrium, although it is useful in a spatially averaged context.

THE PURPOSE of the present note is to discuss an approximate expression for radiative heating within a gas, which employs a radiative response time, and to appraise its applicability through comparison with a more detailed approach. Specific attention is directed to infrared radiation.

In dealing with gaseous systems involving radiative-dynamical interactions, such as planetary and stellar atmospheres, it is desirable to express the radiative heating term in as simple a manner as possible. In this respect, a particularly appealing approximate formulation is

$$\operatorname{div} \mathbf{q_R} = \frac{\rho c_p}{\tau}(T - T_E), \tag{1}$$

where $\mathbf{q_R}$ is the radiative flux vector within the gas, ρ and c_p denote density and specific heat, τ is the radiative response time of the system, T is the local gas temperature, and T_E denotes the local temperature under the condition of radiative equilibrium. The assumed applicability of equation (1) is for small departures from radiative equilibrium.

Equation (1) has, for example, been employed in dynamical analyses of planetary atmospheres by GIERASCH[1] and STONE[2] while GIERASCH and SAGAN[3] have utilized equation (1) for departures from radiative-convective equilibrium. For the terrestrial planets, atmospheric transmission of infrared radiation is due to vibration–rotation bands, and consider for simplicity a single-band spectrum. This would be appropriate for the atmosphere of Mars and the stratosphere of Venus, within which transmission is due to the 15 μ CO_2 band.

The radiative response time, for use in equation (1), is expressed in terms of the total band absorptance by GOODY and BELTON.[4] With reference to the atmospheres of Mars

* This research was supported by the National Science Foundation through Grant No. GK-16755.

and Venus, pressure path lengths are sufficiently large such that the logarithmic asymptote may be employed for the total band absorptance;[5] i.e. the total band absorptance is given by

$$A = A_0 \ln\left(\frac{SPZ}{A_0}\right), \tag{2}$$

where Z is a scaled vertical coordinate, P the pressure of an equivalent homogeneous atmosphere, S the band intensity, and A_0 the bandwidth parameter. Upon employing equation (2) in the expression for τ given by GOODY and BELTON,[4] one has

$$\tau^{-1} = \frac{\pi^2 A_C}{\rho c_p H} \frac{de_{\omega_0}}{dT} \tag{3}$$

with e_{ω_0} denoting Planck's function at the wave number ω_0 of the band, and, as is conventional, the reciprocal wave number of the harmonic thermal perturbation has been replaced by the atmospheric scale height H. Equations (1) and (3) thus describe the approximate radiative flux divergence for transmission due to a single vibration–rotation band.

For purposes of comparison, consider an equivalent homogeneous atmosphere of thickness H, which is bounded by black surfaces having different temperatures. As illustrated by CESS and RAMANATHAN,[5] this is an appropriate atmospheric model, with the temperature of the effective upper surface being controlled through solar absorption by near-infrared bands. In order to induce a departure from radiative equilibrium, it will be assumed that within the homogeneous atmosphere there is a uniform heat source per unit volume Q. Since div $\mathbf{q_R} = Q$, it follows from equations (1) and (3) that the approximate formulation expressing the local gas temperature relative to that for radiative equilibrium is

$$\frac{T-T_E}{QH/A_0}\left(\frac{de_{\omega_0}}{dT}\right) = \frac{1}{\pi^2} = 0.10. \tag{4}$$

Alternatively, an exact solution, corresponding to the total band absorptance of equation (2), may be obtained by superimposing the radiative equilibrium solution of MIGHDOLL and CESS,[6] and the symmetric heat source solution of CESS and TIWARI.[7] Expressing temperature in terms of Planck's function for the single band spectrum, this yields

$$e_{\omega_0}(T) = e_{\omega_0}(T_E) + \frac{QH}{A_0\pi}\sqrt{[\xi(1-\xi)]}, \tag{5}$$

where $\xi = Z/H$, and the origin may be taken at either surface. Now, since small departures from radiative equilibrium are assumed, then $e_{\omega_0}(T)$ may be linearized about $e_{\omega_0}(T_E)$, and equation (5) is recast as

$$\frac{T-T_E}{QH/A_0}\left(\frac{de_{\omega_0}}{dT}\right) = \frac{1}{\pi}\sqrt{[\xi(1-\xi)]}. \tag{6}$$

A comparison of equations (4) and (6) illustrates that the approximate solution, equation (4), does not properly account for local departures from radiative equilibrium. The approximate formulation does, however, appear to be useful in describing average departures throughout the atmosphere. For example, if $\langle T-T_E \rangle$ denotes a spatially averaged quantity, then, from equation (6),

$$\frac{\langle T-T_E\rangle}{QH/A_0}\left(\frac{\mathrm{d}e_{\omega_0}}{\mathrm{d}T}\right)=\frac{[\Gamma(3/2)]^2}{2\pi}=0.125 \tag{7}$$

and equation (4) is in reasonable agreement with this. Indeed, it is in terms of average departures that equation (1) has been employed, for example, by STONE,[(2)] and GIERASCH and SAGAN.[(3)]

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