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An Analysis of the Strong Zonal Circulation within the Stratosphere of Venus[1]

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A dynamical model is presented for the observed strong zonal circulation within the stratosphere of Venus. The model neglects rotational effects and considers a compressible and radiating atmosphere. It is shown that diurnal radiative heating is negligible within the lower stratosphere, a region below 85 km, while observational evidence for the strong zonal circulation pertains to the lower stratosphere within which a direct thermal driving for the circulation is absent. The analysis, however, suggests that propagating internal gravity waves generated by diurnal solar heating of the upper stratosphere induce mean zonal velocities within the upper and lower stratosphere.

Considering the linearized equations of motion and energy, and following Stern's (1971) analysis for an analogous problem, it is shown that the zonal velocity induced by internal gravity waves is retrograde in direction, a result which is in agreement with observation. The nonlinear equations of motion and energy are then solved by an approximate analytical method to determine the magnitude of the zonal velocity. This velocity increases from zero at the tropopause to about 200 m sec^{-1} at the 85 km level. The velocity near the uv-cloud level compares favorably with the observed value of 100 m sec^{-1}.

I. INTRODUCTION

The recent Mariner 10 results (Murray *et al.*, 1974) confirm earlier ground-based observations concerning the retrograde rotation within the atmosphere of Venus (e.g., Boyer, 1973; Carleton and Traub, 1972), as well as interpretations of the Soviet Venera data (Ainsworth and Herman, 1972). Most of these observations pertain to the uv-cloud level, which is located within the stratosphere of Venus between a pressure level of roughly 10 mbar to 40 mbar (Ainsworth and Herman, 1972; Carleton and Traub, 1972). These observations indicate that at least a portion of the atmosphere of Venus is in a state of super rotation with an angular velocity of about sixty times that of the planet; the motion being retrograde and predominately zonal. The principal purpose of the present study is to investigate the phenomenon by which the atmosphere acquires this net angular momentum.

Although models suggesting various mechanisms have been proposed to explain this angular momentum acquisition (e.g., Schubert, 1969; Schubert and Young, 1970; Thompson, 1970; Malkus, 1970), the only analyses which are directly applicable to Venus are those of Gierasch (1970), and Young and Schubert (1973). These are based upon Schubert and Whitehead's (1969) proposal that the retrograde rotation is due to thermal forcing by the periodic overhead motion of the sun. Schubert and Whitehead's model requires a large thermal forcing to be present within the atmospheric region subject to the super rotation, but as will be shown shortly, realistic considerations of the atmosphere

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of Venus reveal that diurnal thermal forcing within the entire lower stratosphere (which contains the uv clouds) is negligible.

We will now describe the response to diurnal radiative heating within the stratosphere of Venus, and for this purpose we adopt the radiative model of Cess and Ramanathan (1972). The model is described in detail by Cess and Ramanathan, and only the results will be summarized here. For the purpose of the present work we shall assume that the atmosphere of Venus consists solely of carbon dioxide. Considering infrared transmission due to the $15\,\mu m$ CO_2 band, while solar absorption results from near infrared bands of CO_2, the net radiative heating is expressed in terms of the dimensionless strong-line parameter

$$\xi = \frac{3\gamma_0 SH}{A_0 d} P^2, \tag{1}$$

where S and A_0 denote the intensity and bandwidth parameter, respectively, for the $15\,\mu m$ band, γ_0 is the mean line half-width, d denotes the mean line spacing, P is pressure and H the scale height.

It has been shown (Cess and Ramanathan, 1972) that most of the near infrared solar absorption is confined to a region for which $\xi \lesssim 1$. To consider the diurnal response of the stratosphere, a parameter ϵ was defined as the ratio of the radiative response time of the atmosphere to the rotation time of the planet. For the upper stratosphere ($\xi \lesssim 1$), ϵ was shown to be equal to 0.084, whereas $\epsilon = 31$ for the lower stratosphere ($\xi \gg 1$). This small value for ϵ for the upper stratosphere indicates a very short atmospheric response time compared to rotation time, and hence large diurnal forcing is to be expected. On the other hand, the large value of ϵ for the lower stratosphere suggests that this region will not respond to diurnal changes imposed by higher atmospheric regions. Based on these results, Cess and Ramanathan (1972) have suggested that a steady-state temperature exists within the lower stratosphere, while the temperature is time dependent but independent of altitude within the upper stratosphere, and these two regions are connected by a transition region.

From (1) it readily follows that $\xi = 4 \times 10^6 P^2$, with P in atm, such that $\xi = 350$ at the 10 mbar level, and clearly the uv clouds are located within the lower stratosphere for which there is no diurnal temperature change. The upper stratosphere, corresponding to $\xi \lesssim 1$, extends upwards from a pressure level of roughly 1 mbar, the altitude of this level being 85 km. Hence diurnal thermal forcing is essentially confined to an atmospheric region above an altitude of 85 km; i.e., well above the uv clouds.

Since the uv clouds constitute the level of the observed super rotation, then from the preceding radiative transfer considerations we conclude that the zonal circulation cannot adequately be explained by the analyses of Gierasch (1970), and Young and Schubert (1973), which are based upon Schubert and Whitehead's (1969) proposal, since these models require a direct thermal forcing within the lower stratosphere. The analyses of Gierasch, and Young and Schubert, employ a diurnal equilibrium profile which has a large day-to-night temperature difference within the lower stratosphere, but this is inconsistent with our previous reasoning. To produce a strong zonal circulation within the lower stratosphere, a mechanism must exist by which diurnal solar heating within the upper stratosphere can induce a circulation within the lower stratosphere.

One very plausible mechanism is the excitation of internal gravity waves within the upper stratosphere, for it is well known from terrestrial tidal theory (Chapman and Lindzen, 1970) that diurnal solar heating excites internal gravity waves, and that these waves can propagate away from the region of excitation into stably stratified regions. The model envisaged is as follows: Diurnal solar heating of the upper stratosphere generates internal gravity waves which propagate away from the region of excitation into the lower stratosphere and upper atmosphere. Propagation of waves results in a net momentum addition within the region subject to solar heating (Stern, 1971), with a corresponding transport of mean horizontal momentum (Booker and

Bretherton, 1967), and consequently the imparting of mean winds, to the lower stratosphere. The strength of the winds depends, among other factors, on the mean energy of the wave. This is expected to be large, since the slow rotation of the planet permits a strong thermal response within the upper stratosphere.

The purpose of the present paper is to investigate the strength of zonal winds induced by the mechanism described above. A similar mechanism for imparting mean motions has recently been proposed by Stern (1971) in a paper intended as a reinterpretation of the moving flame experiment of Schubert and Whitehead (1969). Stern shows that propagating gravity waves can impart mean motions to the fluid. Fels and Lindzen (1974) have also recently considered the generation of mean motions within the atmosphere of Venus by propagating gravity waves. While their calculations do not predict the observed circulation, Fels and Lindzen schematically describe a possible mechanism by which the observed wind speed could develop.

II. Mathematical Model

Restricting the analysis to equatorial regions, the Coriolis force is deleted from the equation of motion, and a two-dimensional cartesian coordinate system is adopted. The Sun is assumed to move in the negative x-direction with a velocity $c = 4\,\mathrm{m\,sec^{-1}}$, where x is a horizontal coordinate measured along the latitude. The hydrostatic assumption is made which permits us to write the conservation equations in pressure coordinates, with $h = -\ln(P/P_0)$, $\omega = dh/dt$, and $\Phi = gz(x,h,t)$, where t is time, P_0 the pressure at the lower boundary of the upper stratosphere, g the gravitational acceleration, and Φ the geopotential.

The appropriate conservation equations for the present study are identical to those derived by Gierasch (1970). The first of these is the Reynold's stress equation

$$\left\langle \omega' \frac{\partial U'}{\partial h} \right\rangle = \frac{\nu}{H} \frac{\partial}{\partial h}\left(\frac{1}{H}\frac{\partial \bar{U}}{\partial h}\right), \tag{2}$$

where $\langle\ \rangle$ denotes a horizontal average, U is the zonal velocity in the x-direction, ν is viscosity, and the field variables have been separated into a horizontal and time averaged part plus a perturbation part; e.g., $U = \bar{U} + U'$. The horizontally averaged energy equation is

$$\left\langle \omega'\left(\frac{\partial T'}{\partial h} + \frac{R}{C_p} T'\right)\right\rangle = \bar{Q}, \tag{3}$$

where T is temperature, R the gas constant, C_p the specific heat at constant pressure, and Q is the net radiative heating per unit mass. Following the same reasoning as presented by Gierasch (1970), the left side of Eq. (3) is small compared to 0(1), such that to first order Eq. (3) yields $\bar{Q} = 0$.

The diurnal radiative heating process has been described in the previous section. To simplify the present analysis, the transition region between the upper and lower stratosphere will be replaced by a transition level which is arbitrarily chosen to be the level for which $\xi = 1$, such that $P_0 = 5 \times 10^{-4}\,\mathrm{atm}$. Denoting the lower and upper stratospheres by regions 1 and 2, the heating functions for the two regions follow from Cess and Ramanathan (1972) to be

$$Q_2 = (\Omega/\epsilon)(\bar{T}_0{}^2/960)[\Xi F(\Omega + x/a) - \phi],$$
$$Q_1 = 0, \tag{4}$$

with

$$\phi = \exp(-960/T)/\exp(-960/\bar{T}_0),$$
$$\Xi = 179\,\delta \exp(960/\bar{T}_0),$$
$$F(\Omega t + x/a) = [\cos(\Omega t + x/a)]^{1/2} \quad \text{for } \cos(\Omega t + x/a) > 0,$$
$$F(\Omega t + x/a) = 0 \quad \text{for } \cos(\Omega t + x/a) < 0,$$

where a is the planetary radius ($a = 6.1 \times 10^8\,\mathrm{cm}$), ϵ is the ratio of radiative response time to rotation time, Ω is the rotational velocity of the planet, δ is the planetary angle factor ($\delta = 4.23 \times 10^{-5}$), and $\bar{T}_0$ is the diurnally averaged temperature for region 2.

Since $\bar{Q}_2 = 0$ from the discussion following (3), then upon averaging (4) it follows that $\Xi = 1/0.382$ (Cess and Ramanathan, 1972), which yields $\bar{T}_0 = 164\,\mathrm{K}$. Recall from the previous section that the temperature within the upper stratosphere is independent of altitude. For the steady state temperature profile within the lower stratosphere (region 1), results obtained from the Mariner 5 occultation experiment are adopted (Fjeldbo *et al.*, 1971). The lapse rate within this region can satisfactorily be represented by a constant value of $dT/dz = -4.5\,\mathrm{K/km}$. The resulting temperature profiles for regions 1 and 2 are illustrated in Fig. 1, while regions 3 and 4 will be explained later.

The remaining equations are the mean field equations which describe the perturbation quantities, and from Gierasch (1970),

$$\frac{\partial U'}{\partial t} + \bar{U}\frac{\partial U'}{\partial x} + \omega'\frac{\partial \bar{U}}{\partial h} = -\frac{\partial \Phi'}{\partial x}, \quad (5)$$

$$\frac{\partial \Phi'}{\partial h} = RT', \quad (6)$$

$$\frac{\partial U'}{\partial x} + \frac{\partial \omega'}{\partial h} - \omega' = 0, \quad (7)$$

$$\frac{\partial T'}{\partial t} + \bar{U}\frac{\partial T'}{\partial x} + \omega'\left(\frac{\partial \bar{T}}{\partial h} + \frac{R\bar{T}}{C_p}\right) = Q'. \quad (8)$$

The viscous diffusion term has been deleted in (5), consistent with the argument of Gierasch (1970). A detailed discussion of the validity of the mean field equations to

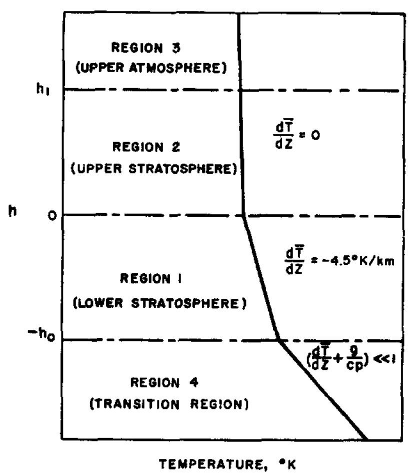

FIG. 1. Thermal-structure model for the atmosphere of Venus. Diurnal variations are restricted to regions 2 and 3, while regions 1 and 4 are in a steady state.

problems involving large mean winds is given by Malkus (1970), and these equations have been used extensively in previous analyses (e.g., Schubert and Young, 1970; and Young and Schubert, 1973) dealing with the Venus circulation. These equations can be justified somewhat if $U'/\bar{U}$ and $\omega'/\bar{U} \ll 1$. The results in Section IV will indeed verify this.

It remains to describe the heating perturbation Q'. This is zero for region 1, and we need consider only region 2. Assuming further that $T'/\bar{T}_0 \ll 1$. then

$$\phi = 1 + (960/\bar{T}_0^{\,2})\,T'$$

within (4), while it is recalled that $\Xi = 1/0.382$. The solar absorption function $F(x,t)$ occurring in (4) is aperiodic, since it vanishes over half the period. However $F(x,t)$ may be separated into periodic terms with n harmonics using Fourier analysis, with the first harmonic term $n = 1$ being the diurnal heating term (Chapman and Lindzen, 1970). Considering only the diurnal term, Q' for region 2 may be written as (Ramanathan, 1973)

$$Q' = \Omega \bar{T}_0 a_0 \,\mathrm{Re}\left|e^{i(\Omega t + x/a)} - (\Omega/\epsilon)T'\right|, \quad (9)$$

where $a_0 = \pi\bar{T}_0/1290\epsilon$, $i = \sqrt{-1}$, and Re stands for the real part of a complex quantity.

From (7) the stream function is defined as

$$U' = e^h(\partial\psi'/\partial h), \quad \omega' = -e^h(\partial\psi'/\partial h). \quad (10)$$

Since $Q' \sim e^{i(\Omega t + x/a)}$, (5)–(8) suggest that ψ', T', and Φ' are separable, such that we may let (Gierasch, 1970)

$$\begin{matrix}\psi' \\ T' \\ \Phi'\end{matrix} = \left\{\begin{matrix}\hat{\psi}'(h) \\ \hat{T}'(h) \\ \hat{\Phi}'(h)\end{matrix}\right\} e^{i(\Omega t + x/a)}, \quad (11)$$

where it is understood that the real part is to be taken, and $\hat{\psi}'$, $\hat{T}'$ and Φ' are complex quantities.

Upon substituting (11) into (5)–(8),

$$(d^2 Y_m/dh^2) + k_m^{\,2}(h) Y_m = F J_m e^{-h/2}, \quad (12)$$

while from (2),

$$\frac{d\bar{V}}{dh} = \frac{S}{4}\,\mathrm{Im}\left| Y_m \frac{dY_m^*}{dh} - Y_m^* \frac{dY_m}{dh}\right|, \quad (13)$$

where $\bar{V} = \bar{U}/c$, the subscript m refers to he region, with $m = 1$ and $m = 2$ denoting egion 1 and 2, respectively, Im stands for 'Imaginary quantity of", and $Y_m{}^*$ is the omplex conjugate of Y_m. Furthermore,

$$Y_m = e^{h/2}\,\hat{\psi}_m{}'/c,$$

$$k_1{}^2(h) = -\frac{1}{4} - \frac{d\bar{V}/dh}{1+\bar{V}} + \frac{\sigma_1 F}{(1+\bar{V})^2},$$

$$k_2{}^2(h) = -\frac{1}{4} - \frac{d\bar{V}/dh}{(1+\bar{V})} + \frac{\sigma_2 F}{(1+\bar{V})(1+\bar{V}-i/\epsilon)},$$

$$J_1 = 0,$$

$$J_2 = ia_0/(1+\bar{V})(1+\bar{V}-i/\epsilon),$$

$$\sigma_m = \frac{1}{\bar{T}_0}\left(\frac{d\bar{T}_m}{dh} + \frac{R\bar{T}_m}{C_p}\right),$$

$$F = R\bar{T}_1/c^2,$$

$$S = H_0{}^2\,\Omega/\nu_0,$$

where $\bar{T}_2 \equiv \bar{T}_0$, H_0 is the scale height corresponding to $\bar{T}_0$ ($H_0 = 3.75$km), and ν_0 refers to $\bar{T}_0$ and P_0. The quantity σ_m is simply the static stability parameter, with $\sigma_m > 0$ signifiying stable stratification while $\sigma_m < 0$ is unstable. S is 2π times the ratio of the viscous diffusion time across a scale height to the period of a thermal wave.

III. Linearized Solution

Equations (12) and (13) are our final equations, and these are mutually coupled through the coefficients $k_m{}^2$ and J_m. We will first identify the wave solution by invoking the physically unrealistic assumption that $\bar{V} \ll 1$, such that $k_m{}^2$ and J_m are constant and (12) is thus linear.

The solution of the homogeneous form of (12) is

$$Y_m = Ae^{+ik_m h} + Be^{-ik_m h}. \qquad (14)$$

If $k_m{}^2 > 0$, then k_m is real and (14) represents a wave solution. Upon combining (11) and (14), the motion is represented by oscillations of the form $\exp[i(\Omega t + x/a \pm k_m h)]$, where $2\pi/\Omega$ is the period of the wave, Ω its frequency, c is the horizontal phase speed and $2\pi a$ is the horizontal wavelength. The quantity $2\pi/k_m$ may be interpreted as the vertical wavelength in units of scale height. These waves have a vertical component of phase variation given by $k_m h$ and hence propagate in the vertical direction.

The horizontal wavelength is of the order of the radius of the planet, such that these waves could be modified by Coriolis forces. Such thermally excited internal gravity waves modified by Coriolis forces are referred to as thermal tides (Chapman and Lindzen, 1970). The slow rotation of Venus and the strong zonal circulation suggests that Coriolis forces will play a minor role compared with inertia forces in the propagation of tides. It may easily be shown (Ramanathan, 1973) that when $\bar{U} \gg c$, the Coriolis effects in modifying the wave structure are negligible. Hence the wave solutions will simply be referred to as internal gravity waves.

Propagating waves transport energy in the vertical direction, and consistent with the present linearization, $\bar{U}$ may be neglected in (5), so that from (11) the vertical flux of energy follows to be

$$\langle\rho\omega'\Phi'\rangle = -c\,\langle\rho u'\omega'\rangle = -\rho_0 c^2\,\Omega\,\mathrm{Im}|Y\,dY^*/dh|. \qquad (15)$$

Upon introducing (14) into (15), it follows that $\langle\rho\omega'\Phi'\rangle$ is positive for the wave given by Ae^{+ikmh}, while for the wave given by Be^{-ikmh}, $\langle\rho\omega'\Phi'\rangle$ is negative. As is conventional (Brooker and Bretherton, 1967), e^{-ikmh} will be referred to as the downward wave. Equation (15) further reveals that the vertical flow of energy gives rise to a mean vertical transport of horizontal momentum, with upward flow of energy resulting in a transport of prograde momentum while downward energy flow results in retrograde momentum.

We will now demonstrate that the mean momentum transported by internal gravity waves can be absorbed by the stratosphere, thus inducing mean zonal velocities within the atmosphere. Within the present linearized context, the coefficients $k_m{}^2$ become

$$k_1{}^2 = -(1/4) + \sigma_2\,F, \qquad (16)$$

$$k_2{}^2 = -(1/4) + \sigma_2\,F/(1 - i/\epsilon). \qquad (17)$$

It is convenient at this stage to describe the various regions within the atmosphere.

Region 2 is bounded above by the upper atmosphere, within which radiative heating is negligible (Dickinson, 1972; Ramanathan and Cess, 1974). This region will be referred to as region 3, and the transition level is taken as $h = h_1$. Region 1 is bounded below by the troposphere, where the lapse rate is nearly adiabatic; i.e., $d\overline{T}/dz + g/C_p \ll g/C_p$. Since the lapse rate within region 1 is strongly subadiabatic, a transition level will be introduced between region 1 and the troposphere which is referred to as region 4. The mean field equations for these two additional regions are the same as before, with

$$k_4^2 = -(1/4) + \sigma_4 F, \tag{18}$$

$$k_3^2 = k_2^2, \tag{19}$$

$$J_3 = J_4 = 0. \tag{20}$$

It remains to specify boundary conditions. The energy flux should either vanish or be a positive finite quantity at the top of the atmosphere, such that

$$\langle \rho\omega' \Phi' \rangle \geqslant 0; \quad h \to \infty. \tag{21}$$

The reason for allowing $\langle \rho\omega' \Phi' \rangle > 0$ is that the wave energy comes from below, and since there are no sources at the top of the atmosphere, the net energy flow should be upwards. This boundary condition is known as the radiation condition (Chapman and Lindzen, 1970). At the boundaries between regions Φ' and ω' should be continuous (Hines and Reddy, 1967), such that

$$dY/dh, \quad Y \text{ continuous at boundaries.} \tag{22}$$

The solution of (12) for the various regions may now be written as

$$Y_1 = A_1(e^{-ik_1h} + R_1 e^{ik_1h}), \tag{23}$$

$$Y_2 = \frac{ia_0}{\sigma_2} e^{-h/2} + A_2 e^{(-k_{2R}+ik_{2I})h} + B_2 e^{(k_{2R}-ik_{2I})h}, \tag{24}$$

$$Y_3 = A_3 e^{(-k_{2R}+ik_{2I})h} + B_3 e^{(k_{2R}-ik_{2I})h}, \tag{25}$$

$$Y_4 = A_4(e^{-ik_4h} + R_4 e^{ik_4h}), \tag{26}$$

where

$$k_m = -(1/4) + \sigma_m F; \quad m = 1, 4,$$
$$k_{2R} = (1/\sqrt{8}) \, (|a_1^2 + a_2^2|^{1/2} + a_1)^{1/2},$$
$$k_{2I} = (1/\sqrt{8}) \, (|a_1^2 + a_2^2|^{1/2} - a_1)^{1/2},$$

with

$$a_1 = 1 - [4\sigma_2 F(1 + 1/\epsilon^2)^{-1}],$$
$$a_2 = 4\sigma_2 F(\epsilon + 1/\epsilon)^{-1},$$

while A_m, B_m, and R_m are constants.

Employing (21) to region 3, then $B_3 = 0$. The lower boundary condition for region 4 is yet to be specified, but the lapse rate within this region is close to adiabatic, and hence we may set $R_4 = 0$ since R_4 is the magnitude of the reflected wave, and the wave will be strongly dissipated on passing through this region. $R_1 e^{ik_1h}$ is the reflected wave due to the change in lapse rate between regions 1 and 4, and thus $|R_1| \leqslant 1$. Furthermore, the top of region 2 corresponds to a pressure of roughly 10^{-3} mbar, or $h_1 \simeq 10$, and it may easily be shown that $B_2 \simeq 0$.

The mean momentum flux, or Reynolds' stress, transported by the wave is $\langle \rho U' \omega' \rangle$ and may be written as

$$\langle \rho U' \omega' \rangle = \frac{\rho_0 c\Omega}{4} \mathrm{Im}\left(Y_m \frac{dY_m^*}{dh} - Y_m^* \frac{dY_m}{dh} \right). \tag{27}$$

Upon combining (27) with (23)–(25), with $B_2 = B_3 = 0$, the Reynolds' stress for regions 1 through 3 follows to be:

Region 1:

$$(2/\rho_0 c\Omega) \langle \rho U' \omega' \rangle = k_1 |A_1|^2 (1 - |R_1|^2). \tag{28}$$

Region 2:

$$(2/\rho_0 c\Omega) \langle \rho U' \omega' \rangle = (a_0/\sigma_2) e^{-(1/2+k_{2R})h} \times \mathrm{Re}\{(A_2/2) e^{ik_2h} - A_2^*(k_{2R} + ik_{2I}) e^{-ik_2h}\} - |A_2|^2 k_{2I} e^{-2k_{2R}h}. \tag{29}$$

Region 3:

$$(2/\rho_0 c\Omega) \langle \rho U' \omega' \rangle = -k_{2I} |A_3|^2 e^{-2k_{2R}h}. \tag{30}$$

From the above results $\langle \rho U' \omega' \rangle$ is a constant within region 1 but a function of h within regions 2 and 3 such that

$\bar{U}/dh)\langle\rho U'\omega'\rangle \neq 0$. The nonzero divergence nplies a net conversion of wave energy, ith (2) illustrating that mean velocities ill be induced within the atmosphere. 'his nonzero divergence of the Reynolds' tress is due to the correlation of the wave eld with heating as given by the terms within the flower bracket in (29) together with the radiation terms as given by the ast term in (29) and by (30).

The direction of the mean flow induced within region 2 is easily shown to be etrograde. An analogous proof has been given by Stern (1971). Considering the horizontal average of the x-component of the momentum equation, and integrating this over region 2, it is found that

$$\frac{\partial}{\partial t}\int_0^{h_1} (\rho \bar{U})dh = \langle \rho U' \omega' \rangle_{h=0} - \langle \rho U' \omega' \rangle_{h=h_1}. \quad (31)$$

For simplicity of presentation let the bottom boundary of region 1 be rigid, for which $|R|_1 = 1$, while the radiation term will be neglected, which corresponds to $k_{2R} = 0$. It then follows from (28), (30), and (31) that

$$\frac{\partial}{\partial t}\int_0^{h_1} (\rho \bar{U})dh = \left(\frac{\rho_0 c\Omega}{2}\right) k_{2I} |A_3|^2. \quad (32)$$

Equation (32) illustrates that retrograde (positive) momentum is added to region 2, and this proof is equally applicable for $R_1 \neq 1$ and $k_{2R} \neq 0$. Stern (1971) gives the following physical explanation: Away from region 2 (i.e., region 3), $\langle\rho U'\omega'\rangle < 0$ such that periodic heating pumps momentum in the negative x-direction within region 3. There is a compensating torque within region 2 which causes a mean velocity in the positive x-direction. Equation (32) shows that $\bar{U}$ increases with time and viscous forces will ultimately become important. The steady state is characterized by

$$\langle \rho U' \omega' \rangle = (\mu/H^2)\,(d\bar{U}/dh) + \text{const.} \quad (33)$$

Since $\langle\rho U'\omega'\rangle = 0$ and $d\bar{U}/dh = 0$ for $h \to \infty$, the constant is zero. Equation (33) then shows that, in a steady state, the Reynolds' stress is balanced entirely by the viscous stress, and the final form of this equation, with appropriate substitutions, is given by (13).

We next consider region 1. Upon substituting (28) into (13),

$$d\bar{V}/dh = (S/2)\,k_1 |A_1|^2\,(1 - |R_1|)^2. \quad (34)$$

It has earlier been shown that $|R_1| < 1$, and hence (34) demonstrates that $\bar{V} > 0$, implying that retrograde velocities will be induced within region 1 also. In deriving (34), it has implicitly been assumed that $(d/dh)\langle\rho U'\omega'\rangle \neq 0$ throughout region 4. Several dissipative mechanisms can be proposed (Ramanathan, 1973) which would give rise to a nonzero divergence of $\langle\rho U'\omega'\rangle$. These mechanisms affect only the coefficient $|R_1|$, and since it will be shown later that the mean velocity is very weakly dependent on $|R_1|$, consideration has not been given to these details in the present linearized description.

IV. Stratospheric Circulation

The previous section indicates qualitatively that mean retrograde winds can be induced within the stratosphere. However, the actual magnitude of the wind can be estimated only by solving the nonlinear mean field equations, and this will be attempted in the present section. Due to the vanishing of radiative heating within region 1, it is found necessary to treat regions 1 and 2 separately. The presence of the heating term within region 2 precludes the possibility of an exact solution of the mean field equations for this region, and correspondingly an asymptotic analysis is presented.

Lower Stratospheric Circulation

The governing mean field equations and the periodic heating function are the same as described in Section II, and the atmosphere is again divided into separate regions, such that the individual regions are solved and matched by appropriate boundary conditions.

With regard to region 1, $k_1{}^2(h)$ appearing in (12) is a function of $\bar{V}$ and $d\bar{V}/dh$, while $\bar{V}$ from (13) is a function of $Y(dY/dh)$, and the two equations are coupled and nonlinear. A closed form solution to these coupled equations, however, has been shown to exist (Booker and Bretherton, 1967; Ramanathan, 1973) for internal gravity waves with a constant mean shear (i.e., constant $d\bar{V}/dh$). The procedure is straightforward and the solutions may be written as

$$Y_1(\eta) = A_1[M_-(\eta) + R_1 M_+(\eta)], \quad (35)$$

$$d\bar{V}/dh = (S/2)|A_1|^2 (1 - |R_1|^2)|\chi|, \quad (36)$$

where

$$\eta = \frac{1 + \bar{V}(0)}{d\bar{V}/dh} + h,$$

$$\chi^2 = 1 - \frac{4\sigma_1 F}{(d\bar{V}/dh)^2} = 1 - 4\mathrm{Ri},$$

with Ri denoting the Richardson number,

$$\mathrm{Ri} = \frac{(g/\bar{T})\,(d\bar{T}/dz + g/C_p)}{(d\bar{U}/dz)^2},$$

while A_1 and R_1 are constants, and $M_\pm(\eta)$ denote the Whittaker functions given by (Abramovitz and Segun, 1965)

$$M_\pm(\eta) = \eta^{(1 \pm i|\chi|/2)} e^{\eta/2}\,{}_1F_1\,(3 \pm i|\chi|/2; 1 \pm i|\chi|; \eta),$$

where ${}_1F_1(\ )$ is the confluent hypergeometric function.

Equation (35) describes a propagating wave, and ${}_1F_1(\ )$ is a real function since η is real, and the phase variation of the wave is adequately described by $\eta^{\pm i|\chi|/2}$, which may in turn be written as $\exp[\pm(i|\chi|/2)\ln\,\eta]$. Based upon previous convention, $\eta^{-i|\chi|/1}$ denotes a downward propagating wave and $\eta^{i|\chi|/2}$ an upward wave. The amplitude of the wave is a function of h, as opposed to the linear case in which the amplitude was constant. The amplitude variation is due to a continuous exchange of energy between the wave and the mean flow.

The reflection coefficient $|R_1|$ will always be less than 1, and thus we conclude from (36) that the motion is retrograde in direction within region 1. The magnitude of the mean velocity may be obtained from (36) once A_1 and R_1 are known. For this purpose it is necessary to obtain solutions for the adjacent regions.

With respect to region 2, the presence of the inhomogeneous term in (12) makes it difficult to attempt an analytical solution. Since we need this solution only to determine the constant A_1, we will adopt a simplified model and assume the velocity to be constant within region 2. Justification for this assumption will be given later.

Since $\bar{V}$ must be continuous at $h = 0$, $\bar{V}$ in region 2 will be equal to $\bar{V}(0)$ as appears within the region 1 solution. In that $\bar{V}$ is a constant within region 2, then $k_2{}^2$ is also a constant and the results of the previous section apply. The solution for Y_2 is given by (24) with $B_2 = 0$ as per prior justification. The only modification concerns the constants a_1 and a_2, which appear in the definitions for k_{2R} and k_{2I}, and these become

$$a_1 = 1 - \frac{4\sigma_2 F}{[1 + \bar{V}(0)]^2}\left\{1 + \frac{1}{\epsilon^2[1 + \bar{V}(0)]^2}\right\}^{-1},$$

$$a_2 = \frac{4\sigma_2 F}{[1 + \bar{V}(0)]^2} \times \left\{\epsilon[1 + \bar{V}(0)] + \frac{1}{\epsilon[1 + \bar{V}(0)]}\right\}^{-1}.$$

Consider next region 4. This region can be expected to have a large apparent viscosity due to turbulence, and hence it will be assumed that a significant mean velocity cannot be induced within this region. With $\bar{V} = 0$, the solution for Y_4 is given by (26) with $R_4 = 0$ as per prior discussion.

It remains to specify boundary conditions at $h = 0$ and $h = -h_0$. One boundary condition is that the mean velocity $\bar{V}$ must be continuous at boundaries, whereas the mean velocity profile assumed for regions 1 and 4 gives rise to a discontinuity in $d\bar{V}/dh$ at boundaries. Booker and Bretherton (1967) have considered exactly this situation. They illustrate that ω' and ϕ' should be continuous at boundaries, which yields

$$\left.\begin{aligned} &Y_1 = Y_m \\ &\frac{dY_1}{dh} - \frac{Y_1\,(d\bar{V}/dh)}{1 + \bar{V}} = \frac{dY_m}{dh} \end{aligned}\right\} h = h_m,$$

with $m = 2,\ 4$; and $h_2 = 0$ and $h_4 = -h_0$. From the preceding considerations, the onstants A_1 and R_1 are obtained as

$$|A_1|^2 = 2\left(\frac{a_0}{\sigma_2}\right)^2 \frac{\sinh(\pi|\chi|/2)}{|\chi|(1+|R_1|)^2} \times \exp\left[-h_0\left(1+\frac{\sigma_2 F}{[1+\bar{V}(0)]^2}\right)\right] \times \frac{\sigma_2 F}{[1+\bar{V}(0)]^2}\left|\frac{(\frac{1}{2}-k_{2R})^2 - k_{2I}^2}{(\frac{1}{2}-k_{2R})^2 + k_{2I}^2}\right|, \quad (37)$$

$$|R_1|^2 = \frac{1+\left(\left[\frac{2k_4}{d\bar{V}/dh}\right]-|\chi|\right)^2}{1+\left(\left[\frac{2k_4}{d\bar{V}/dh}\right]+|\chi|\right)^2}. \quad (38)$$

It is apparent from (38) that $|R_1| < 1$, consistent with previous considerations.

Upon substituting (37) and (38) into (36), we can solve for $d\bar{V}/dh$ and thus $\bar{V}(h)$, employing the boundary condition $\bar{V} = 0$ at $h = -h_0$. It is easily shown that only one solution exists, which corresponds to retrograde motion. Since $\chi^2 > 0$ for propagating waves to be possible, then it follows from the definition of χ^2 that $\mathrm{Ri} > \frac{1}{4}$, which is the condition for shear flow stability. Thus our solution suggests that a shear flow supported by Reynolds' stresses will not lead to a shear flow instability.

To determine $\bar{V}(h)$, we must first evaluate k_{2R}, k_{2I}, χ^2, and k_4, together with the parameters σ_1, σ_2, F, and S. Since $d\bar{T}/dh = 0$ in region 2, then $\sigma_2 = R/C_p$, with $C_p/R = 3.7$. The bottom boundary of region 1 is the tropopause, for which $P \simeq 0.3$ atm, so that $h_0 = 6.4$. In addition, the lapse rate $dT/dz = -4.5\,\mathrm{K/km^{-1}}$ corresponds to $d\bar{V}/dh = -19.5$K, and with $\bar{T}_1 \simeq 200$K, $\sigma_1 = 0.21$. Furthermore $F = R\bar{T}_0/c^2 = 1900$, while $S = H_0{}^2\,\Omega/v_0 = 1.25 \times 10^3$. The constant a_0 refers to the heating term within region 2, with $a_0 = \pi\bar{T}_0/1920\epsilon = 3.14$ for $\epsilon = 0.084$.

Mean velocity profiles are illustrated in Fig. 2 for different assumed lapse rates within region 4, the lapse rate appearing in the expression for k_4. All the profiles show a large retrograde velocity within the lower stratosphere. The uv cloud region is indicated in the figure and the $100\,\mathrm{m\,sec^{-1}}$ wind presumably pertains to this region. The $100\,\mathrm{m\,sec^{-1}}$ velocity in the figure lies well within this region, but what is more important than the actual magnitude is the fact that large velocities can be induced within this region where direct diurnal solar heating is negligible.

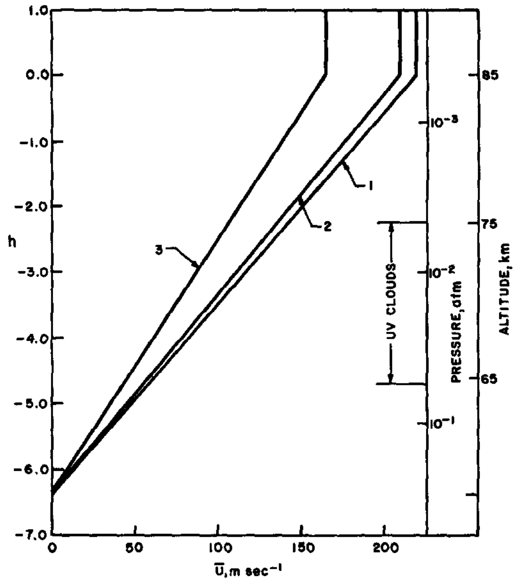

FIG. 2. Mean velocity results for the lower stratosphere. Curve 1: $(d\bar{T}/dz + g/C_p)_4 = 0.1$ K/km; Curve 2: $(d\bar{T}/dz + g/C_p)_4 = 0.01$ K/km; Curve 3: $\nu_e = 10^4\,\mathrm{cm^2\,sec^{-1}}$ for lower stratosphere.

Curve 1 corresponds to $(d\bar{T}/dz + g/C_p)_4 = 0.1\,\mathrm{K\,km^{-1}}$, with $|R_1| = 0.5$, while for curve 2 $(d\bar{T}/dz + g/C_p)_4 = 0.1\,\mathrm{K\,km^{-1}}$ and $|R_1| = 0.96$. It is seen that there is very little difference in the mean velocity profiles. The lapse rate and the dynamics of region 4 affect only the coefficient $|R_1|$, and our justification in not considering the detailed dynamics of region 4 is the insensitivity of the mean velocity profile in region 1 to $|R_1|$.

The molecular kinematic viscosity $\nu_0 = 60\,\mathrm{cm^2\,sec^{-1}}$ was used to determine curves 1 and 2. In the terrestrial stratosphere it is conventional to instead use an eddy viscosity $\nu_e \simeq 10^4\,\mathrm{cm^2\,sec^{-1}}$. Though this value is probably not appropriate to Venus, it was employed to estimate the sensitivity of $\bar{V}$ upon ν_e. Even increasing ν_e by more than two orders of magnitude relative to the molecular value, curve 3 shows that the net effect upon $\bar{V}$ is not large, with the velocity at the 85km level being decreased from $200\,\mathrm{m\,sec^{-1}}$ to $165\,\mathrm{m\,sec^{-1}}$.

We have assumed a constant $\bar{U}$ in region 2 with the justification that this does not affect the results within the lower stratosphere. To verify this, calculations were performed assuming that heating exists only within half a scale height in the upper stratosphere, again yielding a mean velocity of between $165\,\mathrm{m\,sec^{-1}}$ and $210\,\mathrm{m\,sec^{-1}}$ at the 85km level. This intriguing insensitivity of the magnitude of the mean velocity may physically be understood as follows. From (36), $\bar{U}$ varies as $|A_1|^2 = A_1A_1^*$, which is the square of the amplitude of the wave whose source is in region 2 and depends upon the wave structure within region 2 as shown by (37). This dependency is the sole reason for the insensitivity of the mean velocity, and to make the presentation simpler we will neglect the radiation term in k_2 such that A_1 may be written as

$$A_1 = c_0\left(\frac{\frac{1}{2} - ik_2}{\frac{1}{2} + ik_2}\right) \quad \text{for } k_2{}^2 > 0,$$
$$= c_0\left(\frac{\frac{1}{2} - k_2}{\frac{1}{2} + k_2}\right) \quad \text{for } k_2{}^2 < 0,$$

where

$$k_2{}^2 = \frac{\sigma_2 F}{[1 + \bar{V}(0)]^2} - \frac{1}{4}$$

and c_0 is a constant which is not revelant to the present description.

The quantity $A_1A_1^*$ now becomes

$$A_1A_1^*/c_0{}^2 = 1 \qquad \text{for } k_2{}^2 > 0,$$
$$= \left(\frac{\frac{1}{2} - |k_2|}{\frac{1}{2} + |k_2|}\right)^2 \quad \text{for } k_2{}^2 < 0, \tag{39}$$

Now for $k_2{}^2 = 0$, $\bar{U}(0) = 170\,\mathrm{m\,sec^{-1}}$, while for $k_2{}^2 < 0$, $\bar{U}(0)$ exceeds $170\,\mathrm{m\,sec^{-1}}$, and (39) illustrates that $A_1A_1^*$ decreases rapidly. For $k_2{}^2 < 0$, the waves in the upper stratosphere are reflected upwards and only a small fraction of the energy leaks to the lower region. This is signified by the term $(\frac{1}{2} - |k_2|)^2$ in (39). Since $\bar{U}(0)$ varies as $A_1A_1^*$, and since $A_1A_1^* \to 0$ as $\bar{U}(0)$ increases, then $\bar{U}(0)$ cannot increase indefinitely. This property sets an upper limit upon $\bar{U}(0)$ which is roughly $200\,\mathrm{m\,sec^{-1}}$. A lower limit for $\bar{U}(0)$ is about $170\,\mathrm{m\,sec^{-1}}$, since when $\bar{U}(0) < 170\,\mathrm{m\,sec^{-1}}$, $k_2{}^2 > 0$ such that $A_1A_1^*/c_0{}^2 = 1$ and the heating is so strong that $\bar{U}(0)$ will be close to $170\,\mathrm{m\,sec^{-1}}$ in spite of using unrealistically low values of $(1 - R_1R_1^*)$ or a high eddy viscosity. From these discussions, it seems appropriate to conclude that the mean velocities within the lower stratosphere should lie between curve 1 and curve 3 in Fig. 2.

We have permitted discontinuities in wind shears and temperature gradients at the boundaries which separate the various atmospheric regions. Although these discontinuities might introduce spurious reflections, such reflections will not significantly affect our results for the mean velocity, since these reflections influence only the coefficients A_1 and R_1, whereas the previous discussion clearly shows that the mean velocity is quite insensitive to the absolute magnitudes of these coefficients.

It is seen from (36) and (38) that $d\bar{V}/dh$, and thus $\bar{V}$, in region 1 depends upon k_4, and hence region 4 may be of importance in producing retrograde mean flows within region 1. For example (38) illustrates that as $|K_4|$ increases, $|R_1|$ decreases, and (36) correspondingly indicates that $d\bar{V}/dh$ increases. Nevertheless, we wish to reiterate that the results for $\bar{V}$ within region 1 are relatively insensitive to the absolute magnitude of R_1, and this is our only justification for not giving detailed consideration to region 4. Furthermore the physical state of the atmosphere within region 4 (e.g. cloud amount, cloud composition, etc.) is not clearly known from existing observations, and thus a more detailed dynamical modeling of this region of the atmosphere seems unwarranted at the present time.

A final point to be mentioned is the stability of the shear flow within the lower stratosphere. Cess and Harshbardhan (1974) have considered the stability criterion for a radiating shear layer, and they show that for Venus the flow is stable when $\mathrm{Ri} > \frac{1}{4}$; i.e., radiation does not destabilize the shear layer. For the profiles shown in Fig. 2, $\mathrm{Ri} > 5$, indicating that destabilization is unlikely.

Upper Stratospheric Circulation

In the preceding development we assumed $\bar{U}$ to be constant and imposed a value of $\bar{U}(0)$, obtained from the solution for the stratosphere, upon the upper stratosphere. This $\bar{U}(0)$ turned out to be large, and we will now show that the parameter values within the upper stratosphere permit such large mean velocities.

Radiative heating occurs within this region, and the corresponding nonhomogeneous term in the nonlinear mean field equations prevents an analytical solution as was accomplished for the lower stratosphere. Furthermore, attempts to numerically solve a similar set of equations (Young and Schubert, 1973), even after invoking the Boussinesq approximation, have met with little success. We will consequently adopt a procedure similar to that suggested by Gierasch (1970). In the Reynolds' stress equation Gierasch sets $\langle\rho U'\omega'\rangle = 0$, arguing physically that the mean velocity increases until the effect of advection of mean velocity on the thermal field completely eliminates the change of its phase with height. Gierasch (1970) has overestimated the radiative heating and uses an ϵ which increases exponentially with altitude, and as a result he obtains a mean velocity which increases exponentially with h, while we have shown that ϵ is constant.

We will seek to estimate the magnitude of the mean velocity with the proviso that $\langle\rho U'\omega'\rangle = 0$. A partial justification for letting $\langle\rho U'\omega'\rangle = 0$ within the momentum equation as a means for estimating the asymptotic steady state velocity $\bar{U}$ is as follows. Let us consider the wave solution for region 2 as given by (24). For simplicity of presentation we consider only the upward wave given by $A_2\exp[-(k_{2R} + ik_{2I})h]$. From (15) it follows that

$$\langle\rho U'\omega'\rangle \propto k_{2I}(h),$$

while the expressions following (26) show that $k_{2I} \to 0$ for large $\bar{U}$, and in turn $\langle\rho U'\omega'\rangle \to 0$. As the mean velocity increases it rapidly eliminates the phase variation of the wave, and recall that it is the phase variation of the wave which induces mean velocities within the atmosphere. A steady state can thus be characterized by that value of the mean velocity which completely dominates the altitude variation of the phase of the wave.

Young and Schubert (1973) conclude that such an a priori assumption is not justifiable, although they arrive at this conclusion employing their results which show that "the mean velocity profiles obtained using the Reynolds' stress equation for decreasing values of kinematic viscosity do not approach the completely inviscid mean velocity profile." But Young and Schubert do not consider internal gravity waves. Furthermore, as discussed previously, our rationale in letting $\langle\rho U'\omega'\rangle = 0$ is not because of negligible kinematic viscosity, but rather that the phase of the propagating wave vanishes as $\bar{U}$ increases. Since the physics of the model considered in this work is significantly different from the model of Young and Schubert, their conclusions are not applicable to the present analysis.

Young and Schubert (1973) further comment that letting $\langle\rho U'\omega'\rangle = 0$ leads to a physically unrealistic result that the mean velocity depends only upon the stratification and is independent of the radiative heating term. On the contrary, we feel it is quite realistic that in a steady state the mean velocity should depend only upon the stratification, the reason being that radiative heating is sufficiently strong to induce strong mean winds in the atmosphere, but the propagation of waves (which is the means by which velocity is induced within the atmosphere) is possible only when the phase $k_2{}^2 > 0$, and as shown earlier $k_2{}^2 \to 0$ as $\bar{U}$ increases. Hence it is this condition which limits the strength of the mean wind. Since $k_2{}^2$ depends only on stratification and mean velocity, it is not unrealistic that the steady state mean velocity depends only on the stratification of the atmosphere.

We will now use this asymptotic approach and let $\langle\rho U'\omega'\rangle = 0$ within the mean field equation and thus delete the Reynolds' stress equation. A moving coordinate system will be adopted defined by $X = x + ct$, and in the X coordinate system the motion will be steady. The horizontal velocity U in this moving coordinate system has the meaning $U = U_a + c$, where U_a is the actual velocity, and we define the dimensionless variables

$$V = \bar{U}/\bar{U}(0), \quad \Psi = \psi'/U(0),$$
$$\theta = T'/\bar{T}_0, \quad \lambda = X/a.$$

We start with (5)–(8), letting $\partial U'/\partial t = \partial T'/\partial t = 0$ with the understanding that $\bar{U} = \bar{U}_a + c$, cross differentiating (5) and (6) to eliminate Φ', and introducing the stream function. The resulting equations are

$$\frac{\partial}{\partial h}\left(Ve^h \frac{\partial \Psi}{\partial h} - e^h \frac{\partial \Psi}{\partial \lambda}\frac{\partial V}{\partial h}\right) = -\frac{R\bar{T}_0}{[\bar{U}(0)]^2}\frac{\partial \theta}{\partial \lambda}, \tag{40}$$

$$\frac{\bar{U}(0)}{ca_0}\left(V\frac{\partial \theta}{\partial \lambda} - \sigma_2 e^h \frac{\partial \Psi}{\partial \lambda}\right) + \frac{\theta}{\epsilon a_0} = e^{i\lambda}. \tag{41}$$

Equations (40) and (41) permit solutions of the form

$$\begin{matrix}\theta \\ \Psi\end{matrix} = \mathrm{Re}\begin{Bmatrix}\hat{\theta}(h) \\ \hat{\Psi}(h)\end{Bmatrix} e^{i\lambda}. \tag{42}$$

Furthermore, Gierasch (1970) has shown that $\langle \rho U'\omega' \rangle = 0$ implies $\hat{\Psi}/\hat{\Psi}^* = \text{const}$; i.e., $\hat{\Psi} = \Psi_R e^{-i\lambda_1}$, where Ψ_R is real and $\hat{\theta} = \theta_R e^{-i\lambda_1}$ and λ_1 is a constant. Letting $\hat{\Psi} = \Psi_R e^{-i\lambda_1}$, employing (40) and (41), and matching real and imaginary parts

$$\theta_R = a_0 \epsilon \cos\lambda_1, \tag{43}$$

$$V = \left(c\tan\lambda_1/\epsilon\bar{U}(0)\right) + \sigma e^h L, \tag{44}$$

where $L = \Psi_R/\theta_R$ is defined by the nonlinear equation

$$\frac{c\tan\lambda_1}{\epsilon\bar{U}(0)} e^h \frac{dL}{dh} - \sigma_2 e^{2h} L^2 + \left[\frac{R\bar{T}_0}{\bar{U}^2(0)}\right] h = c_1, \tag{45}$$

where c_1 is an integration constant. Equation (45) may be converted into Airy's equation (Miller, 1946), and the solution expressed in terms of Airy's function. There are two solutions, and one of them predicts a $\bar{U}(h)$ which changes sign with altitude. Since the physics of the problem does not allow such a variation, this solution is deleted, and the solution for L is

$$L = -\frac{e^{-h}}{\sigma_2}\left|\tfrac{1}{2} + A^{1/3}\frac{A_i'(\eta)}{A_i(\eta)}\right|, \tag{46}$$

where $A_1(\eta)$ is Airy's function, $A_1'(\eta)$ denotes differentiation with respect to η, and

$$\eta = A^{-2/3}(Ah - c_1\beta_1 + 1/4),$$
$$A = \sigma_2 R\bar{T}_0\left(\frac{\epsilon}{c\tan\lambda_1}\right)^2,$$
$$\beta_1 = \frac{\sigma\epsilon\bar{U}(0)}{c\tan\lambda_1}.$$

There are two constants c_1 and λ_1 to be evaluated such that two boundary conditions are necessary. The boundary condition on mean velocity is that dV/dh be continuous across the boundary between regions 1 and 2, and since the shear within region 1 is a constant, while $V(0) = 1$, then

$$dV/dh = 1/h_0; \quad h = 0, \tag{47}$$

with $h_0 = 6.4$. For the second boundary condition we will let

$$\omega' = e^h L = 0; \quad h = 0. \tag{48}$$

This condition, although an approximate one, is consistent with the mean shear introduced in the lower stratosphere. It was previously shown that when $U(0)$ is large, only a small fraction of the energy leaks into the lower stratosphere implying that $\omega' \simeq 0$ at the boundary. With these two boundary conditions (44) and (46) yield

$$\tan\lambda_1 = \epsilon\sigma_2 R\bar{T}_0/0.22c. \tag{49}$$

All the quantities appearing within (49) have previously been described, such that $\tan\lambda_1 = 4.22$, and (44) yields $\bar{U}(0) = 200\,\mathrm{m\,sec^{-1}}$. Indeed a large velocity is thus induced within the upper stratosphere, and the value for $\bar{U}(0)$ agrees with our previous analysis for the lower stratospheric circulation.

The day-to-night temperature oscillation within the upper stratosphere is illustrated in Fig. 3, together with results which apply in the absence of motion, $\bar{U} = 0$ (Cess and Ramanathan, 1972), and it is

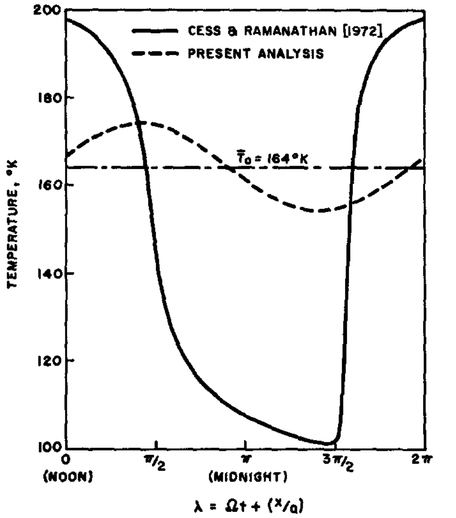

FIG. 3. Diurnal variation of temperature within the upper stratosphere. The curve due to Cess and Ramanathan (1972) is for $\bar{U} = 0$.

seen that the induced motion reduces the maximum temperature difference from 95K to 20K. The maximum temperature occurs at $\lambda = 75°$, while $\lambda = 0$ is the subsolar point. This phase lag is a result of retrograde advection of the hot spot.

The profiles for $\bar{U}$ and $\bar{U}'$ are shown in Fig. 4. It is seen that $\bar{U}'/\bar{U} < 0.1$, satisfying the mean field approximation. For large h, $\bar{U}$ varies as $h^{1/2}$, but the results above 100km may not be valid, since the heating term appropriate to altitudes above 100km is dominated by Doppler broadening, and minor isotopic species contribute to heating as discussed by Dickinson (1972), and Ramanathan and Cess (1974). Hence the applicability of the results should be limited to about 100km. Further, it should be pointed out that the present analysis does not yield the direction of $\bar{U}$. However the linearized analysis indicates that $\bar{U}$ should be retrograde.

V. Conclusions

Strong mean zonal winds are induced by Reynolds' stresses resulting from the phase variation or tilt of convection cells. Though the basic principle, that the strong retrograde circulation for Venus is thermally driven, is similar to the hypothesis originally proposed by Schubert and Whitehead (1969), the dynamical model presented here differs substantially in the mechanism by which Reynolds' stresses are created within the atmosphere. While the viscous diffusion mechanisms proposed by Schubert and Whitehead can generate mean flows, there are several additional features of the problem which lead to the conclusion that this phenomenon does not contribute significantly to the mean winds within the stratosphere of Venus. For example, as illustrated by Gierasch (1970) the viscous term in the mean field equations is negligible when compared with the

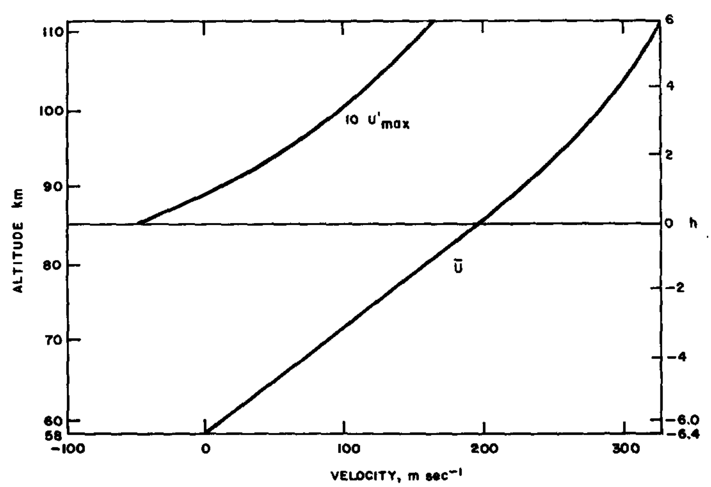

FIG. 4. Fluctuating and mean velocity profiles for the stratosphere of Venus.

inertia term. In addition the stratosphere is stably stratified, and the results presented here as well as the analyses of Stern (1971), and Kelly and Vreeman (1970) reveal the importance of gravity waves in the generation of mean flows. Furthermore the nature of the radiative heating itself supports the mechanism proposed here. Periodic heating is negligible within the lower stratosphere, while the observations of strong motions pertain to the lower stratosphere. As discussed earlier, propagating internal gravity waves are the only apparent mechanism that can induce strong winds in a region within which heating is absent.

The calculated mean zonal wind increases from zero at the tropopause to at least $200 \mathrm{m\,sec^{-1}}$ within the upper stratosphere. The rotational velocity of the planet at the equator is $2 \mathrm{m\,sec^{-1}}$ while the velocity based on a solar day is $4 \mathrm{m\,sec^{-1}}$. The single important reason for this super rotation within the atmosphere is that the slow rotation rate of the planet permits a strong thermal response and a larger phase variation of the wave given by k_m^2.

Another important result from the present analysis is the direction of the zonal flow. The planet is rotating in the retrograde direction and the observations of the atmosphere show that the mean motion is in the same direction. Results presented here regarding the direction of the zonal wind are in agreement with the observations.

The magnitude of the mean winds estimated in the present analysis seems also to be consistent with observation. Boyer (1973) has recently reported a detailed and interesting observation of the zonal winds, and in addition to observing the $100 \mathrm{m\,sec^{-1}}$ wind near the uv cloud level, he has noted irregular clouds circulating more rapidly at higher altitudes with velocities ranging from $100 \mathrm{m\,sec^{-1}}$ to $220 \mathrm{m\,sec^{-1}}$.

Ainsworth and Herman (1972) have deduced velocity profiles from the Soviet Venera data which show a mean velocity of $130 \mathrm{m\,sec^{-1}}$ at the 50km level, and the shear layer extends down to 40km with velocities of $30 \mathrm{m\,sec^{-1}}$ at this level. The lapse rate predicted by Ainsworth and Herman is subadiabatic above 30km, in contrast to the Mariner 5 data (Fjelbdo *et al.*, 1971) which indicates an adiabatic lapse rate below 58km. Nevertheless, if the atmosphere has a subadiabatic lapse rate down to the 30km level, then k_m^2 will be positive and large and the present analysis would predict large mean winds within this region also. Such an attempt was not pursued due to the controversial nature of the data, and also due to the presence of dense clouds at these lower levels.

The mechanism proposed herein for the stratospheric circulation also has implications to the dynamics of the upper atmosphere. As was shown at the end of Section III, waves generated within the upper stratosphere pump prograde momentum to the upper atmosphere (region 3), thereby inducing retrograde winds within the stratosphere. From considerations of conservation of momentum, it follows that prograde velocities should exist within the upper atmosphere. However, the kinematic viscosity is very large in the upper atmosphere, which may severely damp the waves, preventing large prograde winds from being induced within this region. The importance of viscous damping depends, among other things, on the vertical wavelength and on the zonal winds. These two factors are in turn coupled to the magnitude of the viscous damping. The complex nature of this coupled problem defies any simple order of magnitude analysis; it is impossible to be more quantitative on the strength and direction of winds within the upper atmosphere without a detailed analysis of the upper atmosphere. Nevertheless, based upon the present analysis, it is reasonable to conclude that the strength of the retrograde velocity will decrease with increasing altitude in the region above the upper stratosphere, and that the possibility of prograde winds within the upper atmosphere cannot be ruled out.

Two other important aspects of the Venus circulation problem have not been discussed. One concerns the latitudinal transport of zonal momentum, while the second is the variation of the abundance of CO_2 with a four-day period as observed by Young *et al.* (1973). Gierasch *et al.* (1973) have suggested a mechanism for the latter, while the answer to the latitudinal transport problem can only be provided by the solution of the complete three dimensional equations.

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